POWER SYSTEM STABILITY

- Chapter 8
- Weedy & Cory
POWER SYSTEM STABILITY

• DEFINITION
  • The Stability of an interconnected system is it’s ability to return to normal or stable operation after suffering some form of disturbance.

• Instability means loss of synchronism.
  • What is synchronism?
POWER SYSTEM STABILITY

• Power System Stability
  • Steady state stability
  • Transient stability

• Steady state stability
  • determines the upper limit of machine loadings before losing synchronism provided loading is increased gradually.

• Transient stability
  • relates to situations after a sudden and large change of circuit condition (faults or sudden change of load).
POWER SYSTEM STABILITY

• Consider a generating unit connected to an
  infinite bus. (What is an infinite bus?)

\[ I = \frac{|E|/\delta^\circ - |V|/0^\circ}{X/90^\circ} \]

\[ P + jQ = VI^* = \frac{|V|E/90^\circ - \delta}{X} - \frac{|V|^2}{X}/90^\circ \]

• Active power from generator to bus

\[ P = \frac{|E|/|V|}{|X|} \sin \delta = P_{\text{max}} \sin \delta \quad P_{\text{max}} = \frac{|E|/|V|}{|X|} \]
POWER SYSTEM STABILITY

- Active power from generator to bus
  \[ P = P_{\text{max}} \sin \delta \]
  \[ P_{\text{max}} = \frac{|E| |V|}{|X|} \]

- Plot power P versus angle \( \delta \)
- Power angle curve
# POWER SYSTEM STABILITY

<table>
<thead>
<tr>
<th>Power</th>
<th>Motor 0°</th>
<th>90°</th>
<th>180°</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{max}}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
• Maximum power transmitted when angle $\delta$ is $90^\circ$.
• This is the steady state stability limit.
  – Why?

• In practice, generating units are operated at much lower angles.
  – Why?
# POWER SYSTEM STABILITY

- **Equations in Linear and Rotary Motions**

<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>LINEAR MOTION</th>
<th>ROTARY MOTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>(s)</td>
<td>(\theta)</td>
</tr>
<tr>
<td>Mass</td>
<td>(m)</td>
<td></td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>(I = \int r^2 , dm)</td>
<td>(I)</td>
</tr>
<tr>
<td>Velocity</td>
<td>(v = \frac{ds}{dt})</td>
<td>(\omega = \frac{d\theta}{dt})</td>
</tr>
<tr>
<td>Acceleration</td>
<td>(a = \frac{dv}{dt} = \frac{d^2s}{dt^2})</td>
<td>(\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2})</td>
</tr>
<tr>
<td>Force, F</td>
<td>(F \times a)</td>
<td>(F \times r = I \times \alpha)</td>
</tr>
<tr>
<td>Torque, T</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Momentum</td>
<td>(m \times v)</td>
<td>(M = I \times \omega)</td>
</tr>
<tr>
<td>Work</td>
<td>(W = \int F , ds)</td>
<td>(W = \int T , d\theta)</td>
</tr>
<tr>
<td>Power</td>
<td>(P = \frac{dW}{dt} = F \times v)</td>
<td>(\frac{dW}{dt} = T \times \omega)</td>
</tr>
<tr>
<td>K.E.</td>
<td>(\frac{1}{2} m v^2)</td>
<td>(\frac{1}{2} I \omega^2)</td>
</tr>
</tbody>
</table>
MACHINE DYNAMICS

Linear motion
- Momentum = \( m \cdot v \) (newton-sec)
- \( m \) in kg and \( v \) in m/sec
- Kinetic Energy = \( \frac{1}{2} mv^2 = \frac{1}{2} v (mv) \) (joules)

Rotary motion
- Kinetic Energy = \( \frac{1}{2} I \omega^2 = \frac{1}{2} M \omega \)
  - \( I \) = moment of inertia, kg-m^2
  - \( \omega \) = angular speed of rotor (rad/sec)
  - \( M = I \omega \) = Angular momentum (joule-sec/rad)
MACHINE DYNAMICS

• **Rotary motion**
  
  • Kinetic Energy = $\frac{1}{2} I \omega^2 = \frac{1}{2} M \omega$
    
    – $I =$ moment of inertia, kg-m$^2$
    
    – $\omega =$ angular speed of rotor (rad/sec)
    
    – $M = I \omega =$ Angular momentum (joule-sec/rad)

• **Define for a machine**
  
  • $H =$ stored K.E. per MVA of machine = $KE / G$ (MJ / MVA)
  
  • $G =$ Machine rating (MVA)
  
  • $GH = KE = \frac{1}{2} I \omega^2 = \frac{1}{2} M \omega$ but $\omega = 2 \pi f$
  
  • $M = GH / 180 f$ (MJ-sec / deg-elec)

• Note: Unit of KE is now MJ instead of J
POWERS SYSTEM STABILITY

• Inertia constant $H$
  – Between 4 and 9 for condensing turbine generators
  – Between 3 and 4 for non-condensing turbine generators
  – Between 2 and 4 for water wheel generators
    • Note: the unit of $H$ is actually seconds

• Manufacturers still employing FPS system often quote a parameter $WR^2$ in lb-ft$^2$. In such cases compute $H$ from

$$H = \frac{2.31 \times 10^{-10} (WR^2)(RPM)^2}{\text{Machine Rating in MVA}}$$
• Problem 8.1

• An 11 kV 50 Hz 4 pole turbo-generator rated 100 MVA has an inertia constant $H$ of 8 MJ/MVA.
• Find the stored energy in the rotor at synchronous speed. (Ans: 800 MJ)
POWER SYSTEM STABILITY

• SWING EQUATION

Consider a generator

\[ P_{\text{mech}} = \text{Mechanical power input from prime mover} \]
\[ T_{\text{mech}} = \text{Mechanical torque developed} \]
\[ \omega = \text{Rotor speed in electrical rad/sec. Direction same as } T_{\text{mech}} \]
\[ P_{\text{elec}} = \text{Electrical power delivered by generator} \]
\[ T_{\text{elec}} = \text{Generated electro-mechanical torque (opposite to } T_{\text{mech}}) \]
\[ P_{\text{elec}} = \omega T_{\text{elec}} \]

Neglect electrical and mechanical losses. Then

Accelerating torque, \( T_a = T_{\text{mech}} - T_{\text{elec}} \)
POWER SYSTEM STABILITY

- **SWING EQUATION**

  - Accelerating torque, $T_a = T_{\text{mech}} - T_{\text{elec}}$
  - Accelerating power, $P_a = P_{\text{mech}} - P_{\text{elec}}$
  - $P_a = 0$, during steady state operation

  - For motors: $P_a = P_{\text{elec}} - P_{\text{mech}}$

- Force = mass $\times$ acceleration
- Torque = moment of inertia $\times$ acceleration

• **SWING EQUATION**

  - Force = mass × acceleration
  - Torque = moment of inertia × acceleration
  - With θ = angular displacement of rotor w.r.t. a fixed reference.

\[
T_a = I \frac{d^2 \theta}{dt^2} \\
Pa = \omega T_a = \omega I \frac{d^2 \theta}{dt^2} = M \frac{d^2 \theta}{dt^2} \\
M \frac{d^2 \theta}{dt^2} = Pa
\]

POWER SYSTEM STABILITY

- **SWING EQUATION**

\[ M \frac{d^2 \theta}{dt^2} = P_a \]

- Measure rotor position w.r.t a synchronously rotating axis i.e., \( \omega_s \), synchronous speed of the machine.

\[
\frac{d\delta}{dt} = \frac{d\theta}{dt} - \omega_s
\]

\[
\frac{d^2\delta}{dt^2} = \frac{d^2\theta}{dt^2}
\]

\[ M \frac{d^2\delta}{dt^2} = P_a = P_{\text{mech}} - P_{\text{elec}} \quad : \text{Swing equation} \]
• **Problem 10.2**
  
  An 11 kV 50 Hz 4 pole turbo-generator rated 100 MVA has an inertia constant H of 8 MJ/MVA.
  
  If the mechanical input is suddenly raised to 80 MW for an electrical load of 50 MW, find the rotor acceleration. Neglect mechanical and electrical losses.  
  (Ans: 337.5 elect degree / sec\(^2\))
  
  If the acceleration calculated above is maintained for 10 cycles, find the change in torque angle and rotor speed in rpm at the end of this period.  
  (Ans: 28.125 rpm / sec)
STEADY STATE STABILITY

• Definition
  Maximum power that can be transmitted without loss of synchronism.

• Recall the swing equation and the power angle curve.

\[
M \frac{d^2 \delta}{dt^2} = P_a = P_{\text{mech}} - P_{\text{elec}}
\]

\[
P = \frac{|E| |V|}{X} \sin \delta = P_{\text{max}} \sin \delta
\]
Let the system be operating at $\delta_o$.

A small change in $P_{elec}$ occurs due to change of load.

Mechanical input $P_{mech}$ can not change instantly: slow governor.

Generator accommodates the situation by adjusting its power angle.

$P_{elec} \rightarrow P_{elec} + \Delta P$, $\delta_o \rightarrow \delta_o + \Delta \delta$

For small excursions, $\Delta \delta$:

$$\Delta P = \left( \frac{\partial P_{elec}}{\partial \delta} \right)_{\delta_o} \Delta \delta$$

<table>
<thead>
<tr>
<th>Power</th>
<th>P_{max}</th>
<th>$P_{elec} + \Delta P$</th>
<th>$P_{elec}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>$\delta_o$</td>
<td>90°</td>
<td>180°</td>
</tr>
<tr>
<td>$\delta_o + \Delta \delta$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure:** Power angle curve
STEADY STATE STABILITY

$$
\Delta P = \left( \frac{\partial P_{elec}}{\partial \delta} \right)_{\delta_o} \Delta \delta
$$

- Excursions of $\Delta \delta$ is described by

$$
M \frac{d^2(\Delta \delta)}{dt^2} = P_{mech} - P_{elec} = P_{mech} - (P_{elec} + \Delta P) = -\Delta P
$$

$$
M \frac{d^2(\Delta \delta)}{dt^2} + \left( \frac{\partial P_{elec}}{\partial \delta} \right)_{\delta_o} \Delta \delta = 0
$$

$$
\left[ M \frac{d^2}{dt^2} + \left( \frac{\partial P_{elec}}{\partial \delta} \right)_{\delta_o} \right] \Delta \delta = 0
$$

$$
M p^2 + \left( \frac{\partial P_{elec}}{\partial \delta} \right)_{\delta_o} = 0
$$

$$
p = \sqrt{- \left( \frac{\partial P_{elec}}{\partial \delta} \right)_{\delta_o}} = \pm j \sqrt{\left( \frac{\partial P_{elec}}{\partial \delta} \right)_{\delta_o}}\hspace{1cm} M
$$
STEADY STATE STABILITY

\[ p = \sqrt{-\frac{\left( \frac{\partial P_{elec}}{\partial \delta} \right)}{M}} \delta_o \pm j \sqrt{\frac{\left( \frac{\partial P_{elec}}{\partial \delta} \right)}{M}} \delta_o \]

- **Case 1**
  - Roots are imaginary and conjugate. Angle \( \delta \) oscillates about \( \delta_o \) and with decay due to damping action of line resistance and damper windings. So **system is stable**.

\[ \left( \frac{\partial P_{elec}}{\partial \delta} \right)_{\delta_o} > 0 \]

- **Case 2**
  - Roots are real and equal in magnitude. Angle \( \delta \) continues to increase without limit even due to a small disturbance. So system will be unstable under this circumstance.

\[ \left( \frac{\partial P_{elec}}{\partial \delta} \right)_{\delta_o} < 0 \]
Synchronizing coefficient of the machine or stiffness is

\[ \left( \frac{\partial P_{elec}}{\partial \delta} \right)_{\delta_0} \]

stiffness is highest near \( \delta = 0^0 \).
Decrease continuously.
Stiffness is zero at \( \delta = 90^0 \).
Reason generators are operated at an angle much less than 900.
Operation of a machine close \( \delta = 90^0 \) is bad for steady-state stability.
Worse for transient stability considerations.
Problem 10.3

A synchronous generator or reactance 1.20 p.u. is connected to an infinite bus through transformers and a line of combined reactance of 0.6 p.u.

The generator no load voltage is 1.20 p.u. and its inertia constant is 4 seconds.

Line resistance and machine damping is negligible.

System frequency of 50Hz:

Calculate the frequency of natural oscillations when the generator is loaded 50%, 75%, 80% and 90% of its maximum power limit respectively.

(Ans: 0.758 Hz @ 50% and 0.63 Hz @ 80%)
TRANSIENT STABILITY

• Considers the stability of a power system after a sudden change in operating conditions e.g. a short circuit.
TRANSENT STABILITY

• Recall the equations:

\[ P_{elec} = P_{max} \sin \delta \]

\[ M \frac{d^2 \delta}{dt^2} = P_a = P_{mech} - P_{max} \sin \delta \]

• Non-linear differential equation.
• No close form solution is possible.
• Numerical solutions for a short circuit or sudden loss of load
• Swing curve (\( \delta \) versus time) is plotted.
• If \( \delta \) decreases after reaching a peak (\( d\delta/dt = 0 \))
  – Stability is assumed.
• If \( d\delta/dt \) is always positive, system is unstable.
TRANSIENT STABILITY

• Assumptions
• Series resistances and shunt capacitances of transmission lines or generator windings are neglected.
• Effect of damper windings ignored.
• Saliency effects of synchronous machines are neglected. Only direct axis transient reactance is used in the model of machines.
• Voltage behind this reactance is assumed to remain constant.
• $P_{\text{mech}}$ is assumed to remain constant.
TRANSIENT STABILITY

• STABILITY STUDIES
  • Determine whether a system returns to stable operation after a disturbance followed by necessary switching operations.
  • The most serious disturbance to a system is a three-phase fault.
  • A system must be designed to remain transiently stable following a three-phase fault.
  • Fast circuit breakers help restore stability by providing a short clearing time.
TRANSPORT STABILITY

- **EQUAL AREA CRITERION**

- Can be applied to study transient study for a single machine swinging against an infinite bus.
- No numerical solution is involved.
- Remember the swing equation??
- We need to solve this.

\[
\frac{d^2 \delta}{dt^2} = \frac{P_a}{M}
\]
TRANSIENT STABILITY

• Equation to solve:
  \[ \frac{d^2 \delta}{dt^2} = \frac{P_a}{M} \]

• Let \( y \) be a function of \( t \) so that:
  \[ \frac{d}{dt} (y^2) = 2y \frac{dy}{dt} \]

• Set \( y = d\delta/dt \)

• Then:
  \[ \frac{d}{dt} \left[ \left( \frac{d\delta}{dt} \right)^2 \right] = 2 \frac{d\delta}{dt} \frac{d^2\delta}{dt^2} \]

  \[ \left( \frac{d\delta}{dt} \right)^2 = 2 \int \frac{d\delta}{dt} \frac{d^2\delta}{dt^2} \, dt = 2 \int \frac{d^2\delta}{dt^2} \, d\delta = 2 \int \frac{P_a}{M} \, d\delta \]

  \[ \left( \frac{d\delta}{dt} \right)^2 = \frac{2}{M} \int_{\delta_o}^{\delta} P_a \, d\delta \]

  \[ \frac{d\delta}{dt} = \sqrt{\frac{2}{M} \int_{\delta_o}^{\delta} P_a \, d\delta} \]
TRANSIENT STABILITY

- For stability $\frac{d\delta}{dt} = 0$
- Angle $\delta$ must decrease after attaining a peak
- Therefore:
  \[ \frac{d\delta}{dt} = \sqrt{\frac{2}{M} \int_{\delta_o}^{\delta} P_a \, d\delta} \]
  \[ \sqrt{\frac{2}{M} \int_{\delta_o}^{\delta} P_a \, d\delta} = 0 \]
  \[ \sqrt{\int_{\delta_o}^{\delta} P_a \, d\delta} = 0 \]
  \[ \int_{\delta_o}^{\delta} P_a \, d\delta = 0 \]
SINGLE MACHINE CONNECTED TO INFINITE BUS

\[ \int_{\delta_o}^{\delta} P_a \, d\delta = 0 \]

- \( P_e = P_{\text{max}} \sin \delta \).
- \( P_{m0} = P_{e0} = P_{\text{max}} \sin \delta_0 : (a) \)
- \( P_{m0} \to P_{m1} \)
- \( P_a = P_{m1} - P_e : \) rotor accelerates
- \( \omega > \omega_s : \delta \) increases
- At \( \delta = \delta_1 : P_a = P_{m1} - P_e = 0 : (b) \)
- But \( \delta \) still increases as \( \omega > \omega_s \)
- \( P_a < 0: \) \( \omega \) reduces : \( \delta \) still on rise until \( \delta_2 (\omega = \omega_s) : (c) \)
- \( A_1 = A_2 \) for transient stability: \( A_2 > A_1 : \) Unstable.

\[
A_1 = \int_{\delta_o}^{\delta_1} P_a \, d\delta = \int_{\delta_o}^{\delta_1} (P_{m1} - P_e) \, d\delta
\]

\[
A_2 = \int_{\delta_1}^{\delta_2} P_a \, d\delta = \int_{\delta_1}^{\delta_2} (P_e - P_{m1}) \, d\delta
\]

TRANSIENT STABILITY

- For a system to be stable, it should be possible to find angle $\delta_2$ such that $A_1 = A_2$.
- As $P_{m1}$ is increased, a limiting condition is finally reached when $A_1$ equals the area above $P_{m1}$ line.
- Under this condition, $\delta_2$ is maximum:
  \[ \delta_2 = \pi - \delta_1 = \pi - \arcsin \left( \frac{P_{m1}}{P_{\max}} \right) \]
- Any further increase in $P_{m1} \rightarrow A_2 < A_1$
- $\delta$ increases beyond (c)

CRITICAL CLEARING TIME

- There is a maximum value of $\delta$ at which a fault must be cleared to maintain stability.
- Beyond that angle, clearing of fault will not help the system to return to stability.
- This is the so-called critical clearing angle, $\delta_{cr}$.
- The corresponding time is critical clearing time, $t_{cr}$.
- $t_{cr}$ can not be determined by the equal area method.
TRANSIENT STABILITY

- One of two parallel line shorted to ground

- $P_{elec} \neq 0$ but considerably reduced.
- Simplify circuit by one $\Delta$-Y followed by a Y-$\Delta$ conversion.
- Power transfer during fault period is governed by the effective reactance between the two nodes.
- The reactances in parallel with the voltage sources play no part and therefore need not be calculated.
- When the fault is cleared, the power transfer improves due to the lowering of the reactance to the value of the reactance of a single line plus the source reactances.
- Apply equal area criterion:
- It is important to understand the development of $A_1 = A_2$ rather than to depend on the formula.
TRANSIENT STABILITY

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TRANSIENT STABILITY

• Delta-wye conversion

\[ \Delta - Y: \quad Z_A = \frac{Z_{AB} \times Z_{AC}}{Z_{AB} + Z_{BC} + Z_{AC}} \]

\[ Y - \Delta: \quad Z_{BC} = \frac{Z_A \times Z_B + Z_A \times Z_C + Z_B \times Z_C}{Z_A} \]

• Redraw the impedance diagram

• Redraw the impedance diagram.
  – Draw P-δ curve before fault. ($P_{\text{max}}$)
  – Draw P-δ curve during fault. ($r_1 P_{\text{max}}$)
  – Draw P-δ curve after fault is cleared. ($r_2 P_{\text{max}}$)
• Draw the areas $A_1$ and $A_2$ and equate them.
TRANSIENT STABILITY

\[ P \]

\[ P_m \]

\[ \delta_0 \]

\[ \delta_{cr} \]

\[ \delta_{max} \]

\[ P_{max} \sin \delta \]

\[ r_2 P_{max} \sin \delta \]

\[ r_1 P_{max} \sin \delta \]

\[ A_1 \]

\[ A_2 \]

TRANSIENT STABILITY

• The formula:

\[
\cos \delta_{cr} = \frac{(\delta_m - \delta_0)P_{mech}/P_{max} + r_2 \cos \delta_m - r_1 \cos \delta_o}{r_2 - r_1}
\]

• Note: \((\delta_m - \delta_0)\) is in radians, not degrees
important to
Charles Darwin University, 10-May-2010
TRANSENT STABILITY

REFERENCES

• K. Debnath, CDU, 2011.